80 Years of Shear Lag: Historical and Future Perspectives on Volkersen's Insight

Michael D. Bartlett[1] *and David A. Dillard*[2]
[1]*Soft Materials & Structures Lab, Materials Science and Engineering, Iowa State University*
Ames, IA 50011, USA mbartlet@iastate.edu
[2]*Department of Biomedical Engineering and Mechanics, Virginia Tech*
Blacksburg, VA 24061, USA dillard@vt.edu

Introduction

Stress distributions within adhesive layers often determine the performance and durability of structural bonds, prompting significant research and modeling efforts over much of the past century. These analyses have considered an extensive range of joint configurations with varying degrees of sophistication. Since the load-carrying capabilities of many adhesives are often considered better in shear than in tension, shear-dominated joints are frequently employed in structural bond designs, and the shear stress distributions in such configurations are often of significant interest. Perhaps no geometry has received more attention than the widely used single lap joint (SLJ), for which numerous analytical and numerical analyses have been conducted and published. Simple to conduct, single lap joints are thought of as shear-dominated, and with standards (e.g. [3]) purportedly capable of quantifying the "apparent shear strength" (failure load divided by bond area), they are widely used in many industries and institutions. Although peel stresses and adherend plastic bending are more likely to be responsible for the failure of such joints [5], shear stress distributions are also important for a meaningful analysis of such joints, but also for a wide array of other configurations where loads are transferred from one component to another through shear stresses.

Pioneering work by Olaf Volkersen [1] in 1938 is commonly cited for establishing the foundational framework for the shear lag model that is so important for analyzing the shear stress distributions arising when an adhesive joint, composed of two elastic adherends bonded with an elastic adhesive, is loaded in shear. The shear lag model results in a non-uniform shear stress in the adhesive layer, where the stress is highest at the edges and decays towards the center with a characteristic shear lag length determined by the relative stiffnesses and thicknesses of the components. This characteristic length is a metric of the spatial lag required as load is transferred by shear within the adhesive layer from one adherend to the other. Of similar importance to the adhesion community as Winkler's earlier beam on elastic foundation (BoEF) solution [9],Volkersen's shear lag model has been adapted and extended through the years to account for increasingly complex scenarios, allowing his simple framework to continue to provide important insights into adhesive and multi-material systems. Accordingly, Volkersen's analysis has been deployed in a wide range of adhesion and contact scenarios ranging from rigid structural adhesive joints to compliant interfaces, from fiber/matrix interactions to frictional sliding, and from stress analysis to fracture phenomena. Here, we will provide a historical perspective and review of Volkersen's development and discuss how this analysis can be applied to classic and emerging classes of materials and structures. As adhesives, soft materials, and contact sliding continue to find increasing importance in diverse applications, Volkersen's classic work can continue to guide the development of the next generation of materials and structures.

Historical Background

Born in 1907, Olaf Volkersen began his technical career as an intern at Junkers Aircraft and Engine Works, where he gained practical skills and insights into aircraft structures that would mark his long and productive career in the field. After graduating from a technical college in Munich with a specialty in mechanical engineering, he began work with Heinkel Works in 1934. Volkersen's affiliation in the seminal 1938 publication [1] is listed as Ernst-Heinkel-Flugzeugwerke, Seestadt Rostock; the title of the paper, written in German, translates to "The riveting force distribution in tensile stressed rivet joints with constant plate cross sections". Although motivated by interest in riveted joints with discrete attachment points, his idealized representation, involving a smeared continuous interlayer, actually is better suited to model adhesively bonded joints (Figure 1). Neglecting the axial stiffness of the interlayer and shear deformations in the adherends, and assuming linear elastic properties, the shear stress distribution becomes:

$$\tau(x) = A\cosh\omega x + B\sinh\omega x \tag{1}$$

where x is the distance from the center of the bond of length ℓ, $\omega = \sqrt{\frac{G}{h}\left(\frac{E_1 t_1 + E_2 t_2}{E_1 t_1 E_2 t_2}\right)}$,

$$A = \frac{P\omega}{2\sinh\left(\frac{\omega\ell}{2}\right)},\quad B = \frac{P\omega}{2\cosh\left(\frac{\omega\ell}{2}\right)}\left(\frac{E_2 t_2 - E_1 t_1}{E_1 t_1 + E_2 t_2}\right),$$

G and h are the shear modulus and thickness of the adhesive layer, and E_i and t_i are the moduli and thicknesses of the respective adherends.

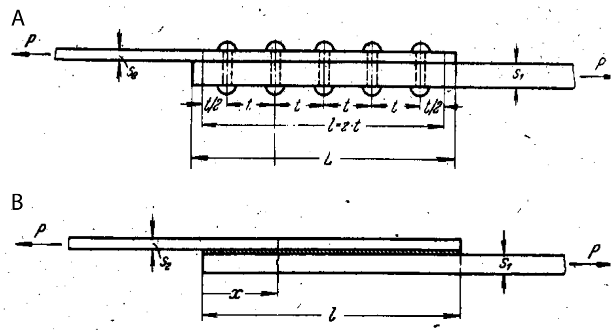

Figure 1. A) Volkersen's rivet formulation and B) his idealized representation, involving a continuous interlayer [1].

As a mechanics of materials rather than elasticity approach, Volkersen's insightful solution does not satisfy all of the boundary conditions for the idealized model, predicting that the shear stresses are maximum at the bond terminations. Whereas shear stresses must in fact vanish on the free surfaces, such details would not have been a concern in his original formulation, which was simply a homogenized version of the discrete rivet problem. Although the analysis is presented for what is effectively a single lap joint configuration, bending moments due to load eccentricity are not considered, though the paper does go on to include symmetric doubler and double lap joint configurations. The SLJ bending effect was subsequently addressed in the seminal work of Goland & Reissner [10], where the moments and transverse shears resulting from eccentricity were determined. They specifically addressed the single lap joint, recognizing the effect of eccentricity and determining the resulting moments and shear. Published six years after Volkersen's paper, they do not reference Volkersen, though their shear stress analysis follows exactly as Volkersen described. The Goland and Reissner analysis finds the shear stresses can be as much as doubled due to the bending moments induced, which locally strain the bondline with additional shear as well as peel stresses, which they also analyzed using a Winkler foundation[9]. Interestingly, their case I elasticity analysis for very stiff "cement" layers does approximate the singular nature of the bondline stresses, including that shear stresses must vanish at bond terminations, though their case II analysis for more flexible interlayers reverts to the mechanics of materials analysis used by Volkersen.

According to his obituary [11] published by the German Aerospace Society, following his death just months before his 100th birthday, Volkersen continued to be very active in the field of aircraft structures. Rising to assume major responsibilities at Heinkel before continuing his education, Volkersen received his doctorate from the Vienna University of Technology in 1944, and went on to serve in several academic positions at the Hamburg School of Engineering and Technical University of Berlin. In 1970, he became Director for Development Planning at VFW-Fokker, which was long a leader in adhesively bonded aircraft.

Extensions and Adaptations

Refinements to and adaptations of Volkersen's shear lag model have been extensive, as the concept has ubiquitous relevance for so many mechanics applications. Without attempting to review the preponderance of these, we simply highlight several areas where this concept has found applications. Volkersen refined his own analysis in a subsequent paper [12], addressing the zero shear stress at bond terminations and including bending effects and peel stress distributions for double lap joints. This paper also includes the analysis of tubular joints subjected to torsion (or panels subjected to in-plane shear), where the Young's moduli of the adherends are replaced by their respective shear moduli, effectively shortening the shear lag distance and increasing the stress concentrations by as much as $\sqrt{3}$. In this paper, several works predating his 1938 publication are cited, apparently addressing shear lag transfer mechanisms, though not for adhesive bonds.

Among the many extensions of the shear lag model are the significant contributions of Hart-Smith, including his modifications made to address inelastic behavior in the adhesive layer. His assumption of elastic-perfectly plastic adhesive behavior resulted in altered solutions that have implications, including for a recent analysis of frictional sliding of extensible strips [13]. Another intriguing application is that of a fiber embedded in a matrix, which has many applications to fiber-reinforced composites, reinforcing rods in concrete, and biological materials. The photoelastic image below shows the shear stress field surrounding a broken glass fiber embedded in a transparent matrix, for example (Figure 2). The shear lag model even becomes the basis for

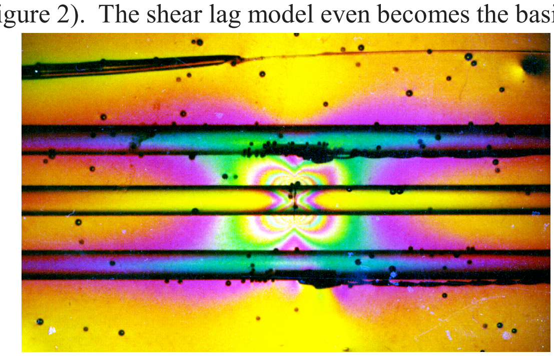
Figure 2. Photoelastic image illustrating shear stresses surrounding a broken fiber in a model composite. [Courtesy: Scott Case and Jack Lesko.]

a method to measure Poisson's ratio to very high precision for elastomeric materials[14, 15], where the compressibility of a confined elastomer results in classic shear lag behavior.

The shear lag model lends itself to a fracture mechanics perspective quite nicely, though many of the analyses do not make explicit use of the shear lag stress distribution, as they focus on self-similar or steady state scenarios, where the details of shear stresses are not required. Examples of this include the Brussat et al analysis of the cracked lap shear joint [16], the Gent el al analysis of cord rubber pullout [17], as well as a host of coating and delamination problems as reviewed in [18]. Fracture analyses including the Volkersen stress field have been employed for shearing of adhesive tapes[19] and the debonding and frictional withdrawal of a fiber embedded in a matrix[20, 21].

Future perspectives

With the importance of stress in bonded joints, Volkersen's analysis provides insight to enable future applications. As seen in Figure 3, we anticipate that fields ranging from advanced adhesives and bio-inspired composites to 3D printing and biomedical applications are areas where future materials and structures can be informed by Volkersen's contributions. These fields represent areas where soft materials come together at rigid interfaces and material properties and geometry of the interface can be designed to display desired function. For example, composite materials inspired by nacre where rigid plates are dispersed in a compliant phase, parameters such as platelet modulus and size can designed through Volkersen's approach or similar analysis. This can be further extended to process control parameters for 3D printing, interfaces in biomedical implants and tissue repair, and geometry and material properties in reversible adhesive interfaces.

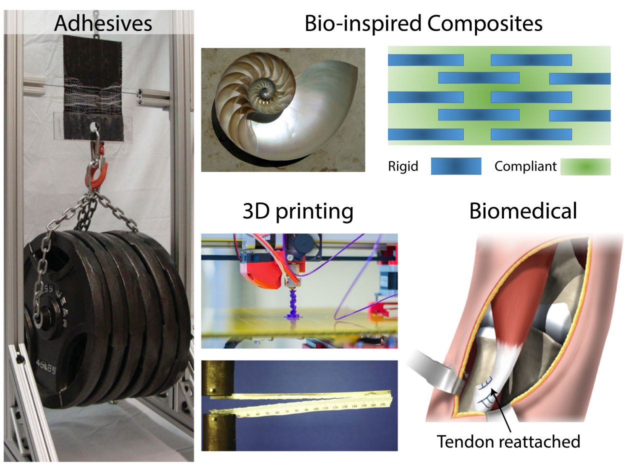

Figure 3. Future research areas with potential to benefit from Volkersen's analysis such as adhesives [2], bio-inspired composites [4], 3D printing [6, 7], and biomedical applications [8].

Conclusions

This presentation is meant as a tribute to Olaf Volkersen and a recognition of the significance of his 1938 shear lag formulation for the field of adhesion science. The myriad applications of this powerful insight have left a pronounced effect on our field.

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